ON THE CALCULATION OF NON-LINEAR AERODYNAMIC CHARACTERISTICS AND THE NEAR VORTEX WAKE*.

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Abstract

This investigation presents methods for the calculation of the distribution of vortices on the wing planform and on the trailing vortex wake by iterative procedures based on the application of the vortex lattice method concepts. In the case when the trailing vortices are taken to leave the wing at the trailing edge only the calculation results in determining the linear aerodynamic characteristics and the shape of a rolled up wake. The present investigation considers the cases when the vortices from each cell are allowed to leave the wing planform at a fixed angle, and the case when the vortex shedding can be limited to the planform edges only. In these cases non-linear aerodynamic characteristics are evaluated. The corresponding trailing vortex wakes were first calculated by using discrete ideal vortices. It is now proposed to use a finite core vortex model thereby eliminating some of the numerical problems associated with the use of the ideal vortices.

List of Symbols

AR	aspect ratio
b	wing span
c_i	local chord
c	chord - rectangular wing
C_ℓ	two-dimensional lift coefficient
C_L	lift coefficient
C_{L_α}	lift slope coefficient
C_M	moment coefficient
h	distance between a point and the line vortex
[H]	geometry influence matrix
k	vortex strength
[K]	vortex strength matrix
ℓ_x, ℓ_y	distance from bound wing vortex cell to a line passing through the wing apex
M	number of steps in the vortex interactions calculation
N	number of discrete vortices used in the vortex sheet representation
V_j	component of the induced velocity (in the three dimensional case) at point (j)
x,y,z	space coordinates (the origin at the mid span trailing edge)
$x_{c.p.}$	longitudinal center of pressure
$y_{c.p.}$	lateral center of pressure
S	wing area
U	free stream velocity
v_i	components of the induced velocity at point (i) (in the two-dimensional case)
[V]	velocity matrix
α	angle of attack
α_i	induced angle of attack
β_{1_i}, β_{2_i}	angles formed between vortex segment and a point in space
Γ	circulation

Subscripts

c	chordwise subdivision index
s	spanwise subdivision index

I. Introduction

The knowledge of the trailing vortex wake is required because of the increasing importance of interactions between aircraft in crowded airlanes and the need to minimize these interactions by accelerating the dissipation of the trailing vortices. A starting point for the calculation of the trailing vortex wake is the realistic evaluation of the distribution of vortices on the wing and in the near wake. The calculation of a realistic distribution of vortices on the wing planform and its trailing wake leads to an additional important result; the accurate evaluation of wing aerodynamic characteristics including the non-linear effects.

The lift of the wing can be calculated by adding the contributions of the vortices distributed over the wing planform and its wake. The classical calculation of lift distributions based on superposition of horseshoe type vortices are summarized in the method presented by Weissinger[1] for evaluating the lift integral equation on the planform of high aspect ratio wings. The case of wings of low aspect ratio was presented by R.T. Jones[2] and Bollay[3]. The representation of the vortex distribution over the high aspect ratio wing planforms by vortex lattice method has been first developed by Falkner[5,6,7] and more recently extended and formulated for numerical computation by Hedman[8], Kalman, Giesing and Rodden[9] and Margason and Lamar[10]. The calculation of lift distribution on low aspect ratio wings including effects of leading edge vortex separation were presented by Brown and Michael[11], Mangler and Smith[12,13,14], Gersten[15] and Polhamus[16]. The calculations of the trailing wake behind the lifting surfaces based on lift distribution which is independently evaluated on the wing planform were presented by Kaden[17], Westwater[18] and Sprieter and Sacks[19]. The representation of the trailing vortex sheet by discrete line vortices was first presented by Westwater[18]. This calculation utilizes the theorems of Betz[20]. Further calculations using discrete line vortices were later presented by Rogers[21], Hacket and Evans[22], Butter and Hancock[23] and Clements and Maull[24]. Additional vortex wake calculations have been presented by Brown[25], Donaldson et al.[26] and Nielsen and Schwind[27].

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Recent investigations have unified the treatment of the vortex distributions over the wing planforms and their wake. These methods aim at obtaining both realistic aerodynamic characteristics of the wing as well as the proper trailing vortices in the wake. Investigations based on this approach are in progress in number of research centers and some of the results are presented in reports by Rehbach[28], Maskew[29], Labrujere[30], Hedman[31], Lind[32] and Rom and Zorea[33].

The present investigation is based on the application of the vortex lattice method concepts. The wing is divided into cells. A bound vortex element is placed at the "quarter chord" of each cell and a control point is placed at the middle of its "three quarters chord" line. The trailing vortices can be taken to be shed away from the edges of each bound vortex and to continue in the wing plane to the trailing edges. On the other hand these vortices can be taken to leave the wing plane at each cell or to leave from the side and trailing edges only. When the vortices are shed from the trailing edge the rolling up of the trailing wake is calculated and linear aerodynamic coefficients are evaluated by an iterative procedure[33]. In the case when the trailing vortices are allowed to leave the wing plane at each cell at an arbitrary local angle (similar to Gersten's[15] model) non-linear aerodynamic wing characteristics are obtained[33]. When the trailing vortices are allowed to leave from the side and trailing edges only the trajectories of these trailing vortices are determined by an iterative calculation which also results in the evaluation of non-linear aerodynamic characteristics.

The numerical problems involved in approximating the circulation distribution by discrete ideal vortices are discussed by Moore[34] and by Rom and Zorea[33]. It is shown in Ref. 33 that the trajectories of the vortices can be calculated under certain conditions when the vortices are shed from the wings' trailing edge only. The numerical difficulties associated with using discrete ideal vortices become even more difficult when the vortices are shed from the cells on the wings' planform or from its side edges. In order to eliminate some of these difficulties it was proposed in Ref. 35 to use the finite core vortex model. As a first approximation the core size is related to the local vortex strength by the value calculated by Sprieter and Sacks[19]. We assume in the present analysis that the influence of the vortices which are generated at the side and trailing edges of the high aspect ratio wing should be similar to those which are generated by the leading and trailing edges of the low aspect ratio wing. Therefore, the present calculation which involves vortices shed from the wings' edges and included the effects of the vortices in the trailing wake is a unified method for the calculation of the lift of any planform in subsonic flow. In this method the strength of the vortices as well as their trajectories are obtained by an iterative procedure in which the wing is represented by a vortex lattice and the trailing vortex wake is represented by discrete ideal vortices or by the finite core vortices. Results of some calculations for non-linear as well as linear aerodynamic characteristics of some wings and their corresponding trailing vortex wakes calculated by the present procedure are included in this paper.

II. The Non-Linear Vortex Lattice Method

This method is based on the division of the wing planform into a lattice of cells. A Horseshoe vortex is placed in each cell with the bound vortex at the 1/4 chord line of the cell. In the linear vortex lattice method[5,6,7,10], the trailing vortices are straight lines embedded in the wings' planform and are shed away into the wake from the trailing edge of the wing. The vortex strength is determined so that the velocity boundary condition at the point in the middle of the 3/4 chord of each cell is satisfied.

The induced velocity field is determined by the Biot-Savart relations:

$$V_j = \sum_{i=1}^{n} \frac{k_i}{4\pi h_i} (\cos\beta_{1_i} + \cos\beta_{2_i}) \quad n \neq j \tag{1}$$

The induced velocity field and the circulation distribution at the various cells are related by the matrix relation

$$[V] = [H]\,[K] \tag{2}$$

where $[H]$ is the influence matrix, determined from the wing planform and its subdivision.

The aerodynamic characteristics such as the lift and the pitching moment as well as the center of pressure may be evaluated from the calculated $[K]$ matrix, by the following relations:

$$C_L = \frac{1}{S} \int_{-b/2}^{b/2} c_i C_{\ell_i}\, dy = \frac{dy}{U.N_s.S} \sum_{i=1}^{N_s} \sum_{j=1}^{N_c} K_{ij} \tag{3}$$

where

$$\sum_{j=1}^{N_c} K_{ij} = \Gamma_i = \frac{U c_i C_{\ell_i}}{2} \tag{4}$$

For rectangular wing

$$C_L = \frac{2}{c.U.N_s} \sum_{i=1}^{N_s} \sum_{j=1}^{N_c} K_{ij} \tag{5}$$

The pitching moment can be estimated by

$$C_M = C_L \frac{x_{c.p.}}{c} \tag{6}$$

where $x_{c.p.}$ is measured from the wings' leading edge,

$$x_{c.p.} = \frac{\sum_{i=1}^{N_s} \sum_{j=1}^{N_c} C_{\ell_{ij}} \ell_{ij} x_{ij}}{C_L} = \frac{\sum_{i=1}^{N_s} \sum_{j=1}^{N_c} K_{ij} \ell_{ij} x_{ij}}{\sum_{i=1}^{N_s} \sum_{j=1}^{N_c} K_{ij}} \tag{7}$$

$$y_{c.p.} = \frac{\sum_{i=1}^{N_s} \sum_{j=1}^{N_c} C_{\ell_{ij}} \ell_{ij} y_{ij}}{C_L} = \frac{\sum_{i=1}^{N_s} \sum_{j=1}^{N_c} K_{ij} \ell_{ij} y_{ij}}{\sum_{i=1}^{N_s} \sum_{j=1}^{N_c} K_{ij}} \tag{8}$$

In order to evaluate the induced drag we start by calculating the induced angle of attack using the matrix equation:

$$[\alpha_i] = [K][H_1] \qquad (9)$$

where the $[K]$ matrix is the one obtained in the lift calculation and $[H_1]$ is a modified influence matrix. The modified matrix is obtained when the control point is positioned at the middle of the bound vortex which is at the 1/4 chord line of the cell.

The spanwise distribution of the induced drag is calculated by the procedure presented in Ref. (35). The induced drag of the wing is then determined using the following relations:

$$C_{D_i} = \frac{1}{S}\int_{-b/2}^{b/2} C_{\ell_i} c_i \alpha_i \, dy \qquad (10)$$

Since

$$c_i C_{\ell_i} = \frac{2}{U}\sum_{j=1}^{N_c} K_{ij} \qquad (11)$$

The induced drag coefficient (for a rectangular wing) is then

$$C_{D_i} = \frac{1}{SU}\sum_{i=1}^{N_s}\sum_{j=1}^{N_c} K_{ij}(\alpha_i)_{ij}\frac{b}{N_s} \qquad (12)$$

The non-linear variation of the aerodynamic coefficient with angle of attack is calculated using either of these flow models:

(1) Following Gersten's theory, we assume that the free vortices leave each cell at an angle of $\alpha/2$ and remain straight in the wake downstream of the wing. The matrix equation is solved in a procedure similar to that used in the calculation of the linear coefficients. In this case the effects of the induced velocities due to the vortices positioned above the wing planform results in non-linear variation of the aerodynamic coefficients.

(2) Flow visualization of the vortices on low aspect ratio wings show that these vortices seem to be shed from the planform edges. Therefore, it is assumed in this calculation that all vortices from the inner cells remain imbedded in the wing plane. Only the vortices from the cells which are positioned at the planform edges are allowed to leave away from the planform into the flow. The local angles of the vortices shed from the wing-edges are determined from the modified vortex lattice calculation following an interative procedure. The rolling up of the vortices shed from the side edges has considerable effect on the calculated values of the aerodynamic coefficients.

III. Calculation of the Vortex Wake

The development of wake calculations starting with the simplified Westwater model and proceeding to more realistic representations are shown graphically in Figs. 1 to 4. The simplied procedure presented in Figs. 1 to 3 have been described in Ref(33)and(35),and only a brief outline is presented here.

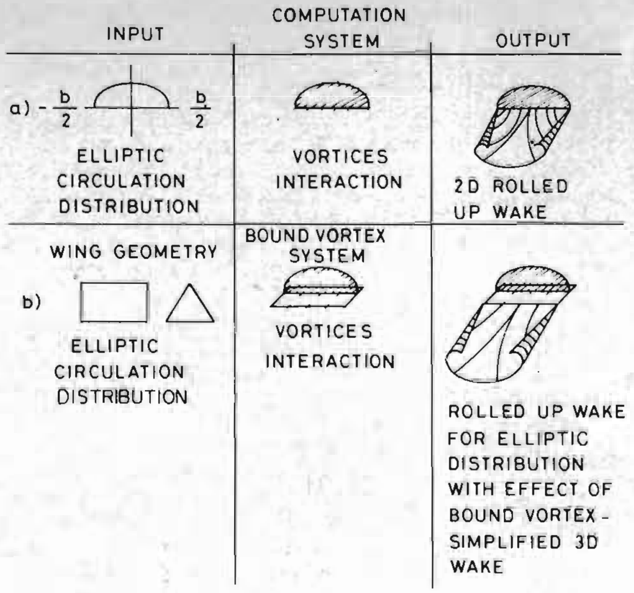

Figure 1. Vortex Wake Calculations for Elliptic Lift Distribution.

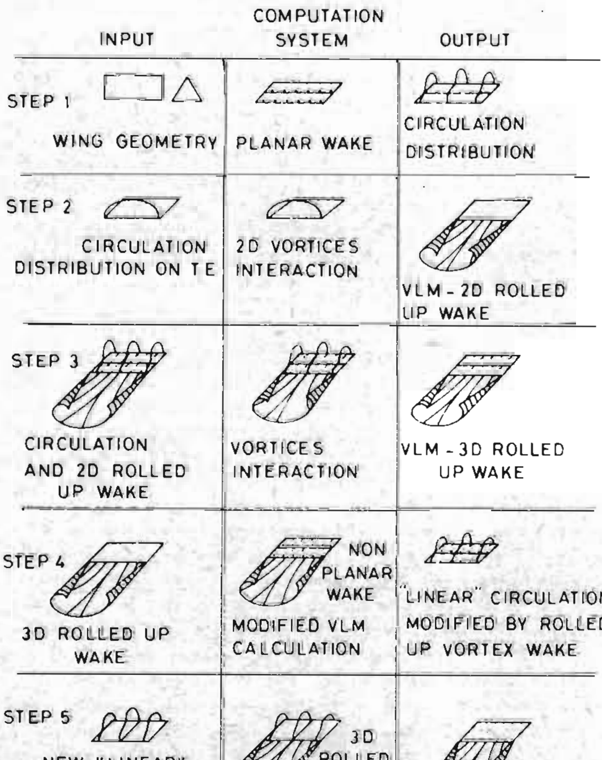

Figure 2. VLM Vortex Wake Calculations.

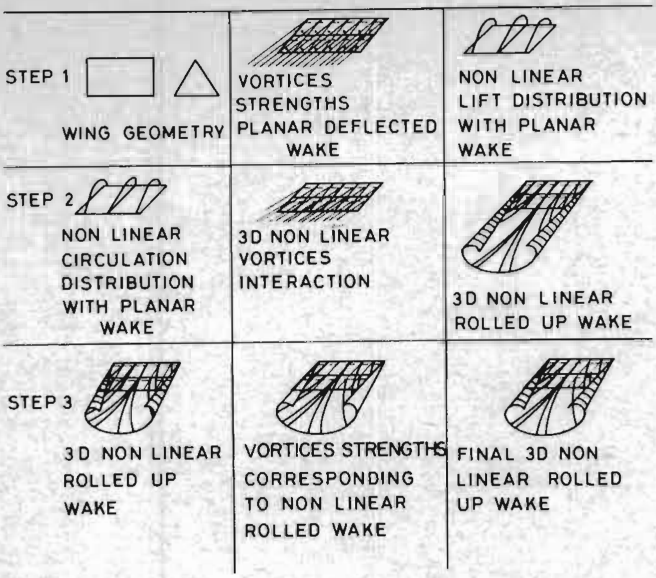

Figure 3. Nonlinear Vortex Wake Calculations.

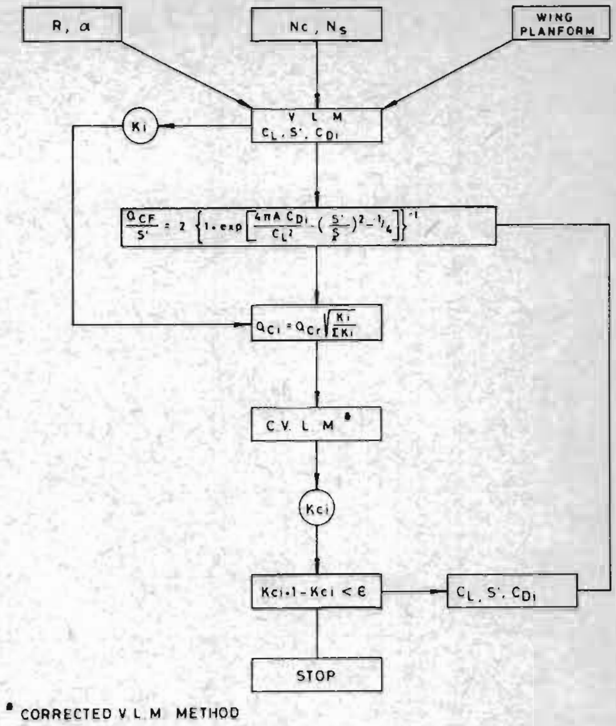

Figure 4a. Schematic Diagram for the Calculation of Cores diameters.

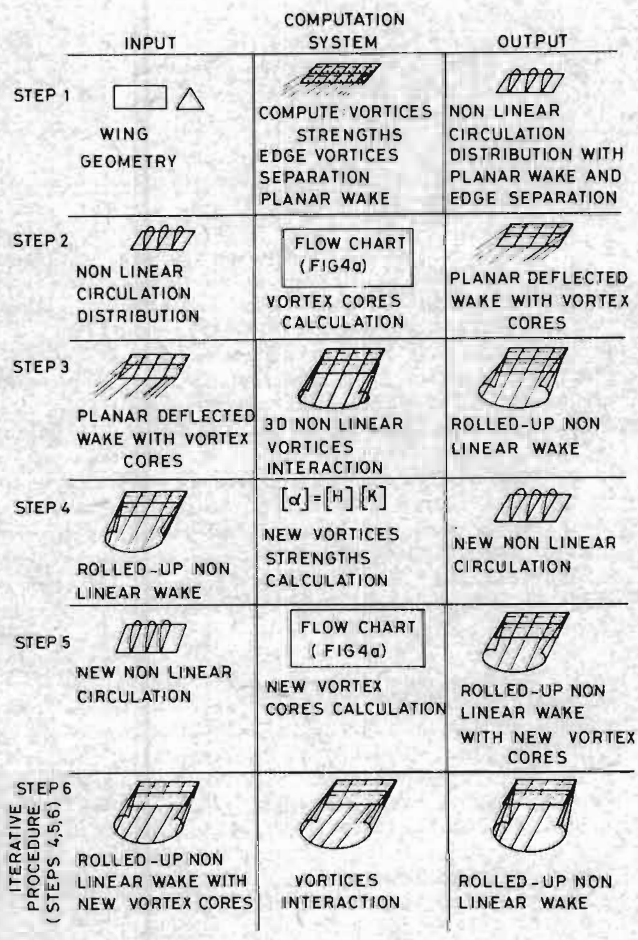

Figure 4. Non-linear Finite Core Rolled up Wake Calculations.

<u>Wakes with Vortices Separating from the Trailing Edges.</u>

(a) <u>Two-Dimensional Wake Model with Elliptic Spanwise Circulation Distribution.</u>

This program reproduces the early work of Westwater[18] in which the flow in a fixed stationary plane far behind a moving wing with elliptic circulation distribution is identified with the problem of the unsteady motion of a two-dimensional array of point vortices moving under their own mutual influences (see (a) in Fig. 1).

This basic program enables us to investigate many of the practical problems which occur in all discrete vortex models: whether to use equal-strength or equally-spaced vortices, the problem of"cross-over" of the vortex trajectories, "escape" of the tip vortex and the development of irregular shape of the sheet (see Figs. 5,6, and 7).

These problems are basically due to the use of a concentrated point vortex model with its associated velocity singularities, plus accumulative effects of rounding-off errors etc. Reasonable numerical solutions, in spite of these difficulties, were found to be possible by a proper choice of the number and spacing of the vortices and the size of the time intervals (correspodnign to the downstream separation of the planes at which the vortex pattern is calculated step by step). The correct combinations were found by a series of numerical experiments. It is important to note that due to the causes mentioned, too many vortices or stations can lead to deterioration of the results rather than convergence to a limiting form.

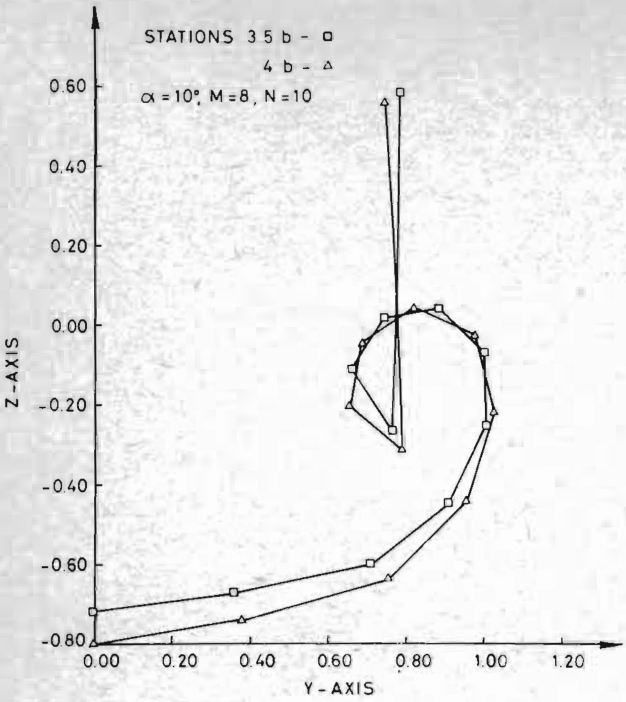

Figure 5. Problems in the Calculations of the Vortex Sheet Roll-up-"Escape" of the "Tip" Vortex.

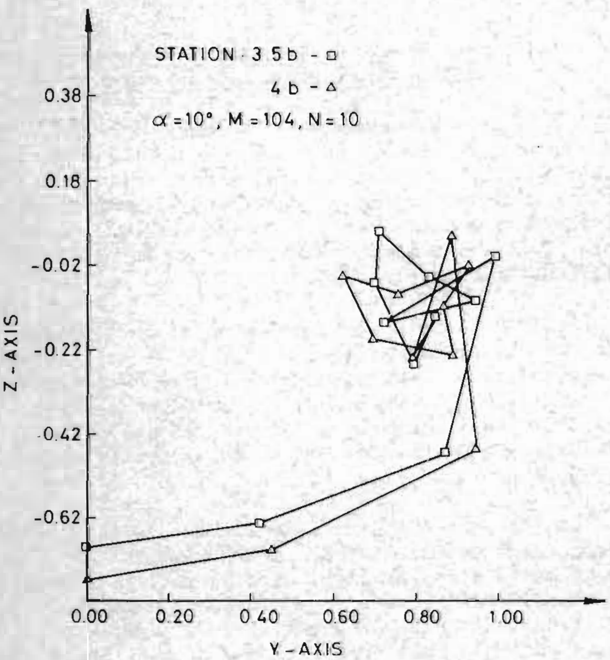

Figure 6. Problems in the Calculations of the Vortex Sheet Roll-up-Cross-Over of Vortex Lines.

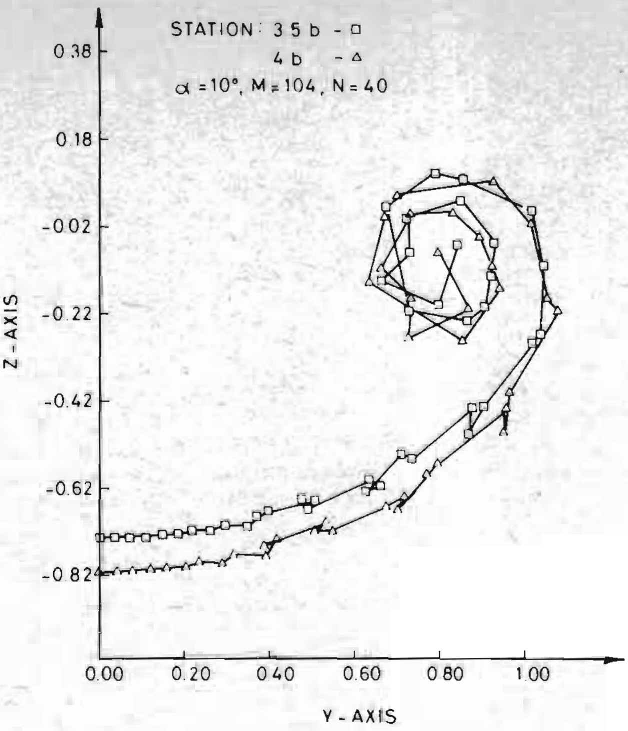

Figure 7. Problems in the Calculations of the Vortex Sheet Roll-up-too Many Sub-divisions.

(b) Lifting-Line Model with Elliptic Spanwise Circulation Distribution.

In the next stage of the work a wing with an elliptic circulation distribution is dealt with, employing a single bound vortex in the wing quarter-chord line (see (b) in Fig. 1). The trailing vortices are continued straight from this line to the trailing edge and, from the trailing edge back-wards. Their rolling up is taken, as a first approximation, to be given by the results in Section (a). The corrections necessary to this wake shape to conform with the lifting-line vortex pattern are then calculated by a series of iterations, assuming the elliptic spanwise circulation distribution un-changed throughout. The experience gained in stage (a) is utilized to select the number of vortices etc. At a later stage the finite-core concept is incorporated here to.

A practical point worth mentioning is as follows. The rolling-up calculation was only carried as far as ten semi-span downstream of the trailing edge. However, a correction for the influence of the portion of the wake downstream of this station was incorporated. At the ninth and tenth stations the center of gravity of the vortex-wake cross-section (in the half-span domain) was found and a line through these two points was taken to define the position and direction of a single replacement trailing vortex for each half wing, starting from the tenth station extending to infinity and having the appropriate ultimate strength (i.e. equal to the value of the mid-span circulation). The induced velocity due to this pair of semi-infinite vortices was taken as an appropriate correction for the influence of the portion of wake downstream of the tenth station on the flow upstream of it. This method was employed in all subsequent stages.

Result of calculation of the wake shape behind a rectangular wing of AR = 3 is shown in Fig. 8. A comparison of this result with that of the infinite vortices (Westwater model) is indicated as the 2-D curve on this Figure.

the vortices distribution on the wing and the aerodynamic coefficients are recalculated. We then return to the recalculation of a new vortex wake shape by iterations, keeping the vortex strengths fixed, and so on. Some wake shapes calculated by these procedures are shown in Figs. 9 and 10.

(d) Models with Vortices Leaving the Planform

The calculation starts with the free vortices leaving at an angle of $\alpha/2$ to the wing planform and then extending in straight lines into the wake.

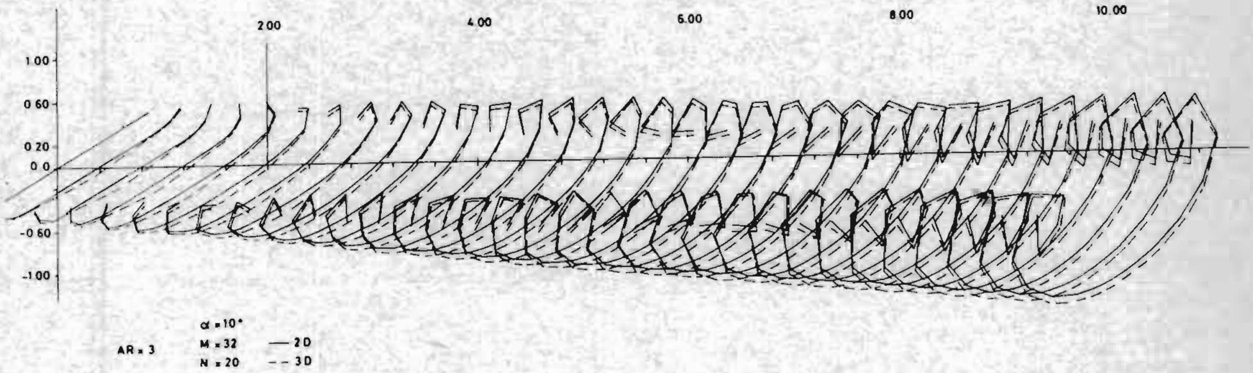

Figure 8. The Two and Three Dimensional Rolled up Wake Shapes Calculated by Elliptic Lift Distribution Procedure. (Axes are Given by Half Span Units).

(c) Model Employing a Single Sheet Vortex Lattice.

At this stage a lifting-surface theory was introduced to enable us to deal with general wing shapes with some accuracy (Fig. 2).

The standard vortex-lattice technique is employed[5-10] to start with. The wing is divided into cells by a number of equally spaced streamwise lines and by either equally-spaced constant-percentage-chord lines or by lines perpendicular to the stream. A bound vortex element is placed at the "quarter-chord" of each box and a control point is established at the middle of its "three-quarter-chord" line. The usual trailing vortices are taken to spring from each bound vortex element and to continue straight downstream in the wing plane, and the unknown vortex strengths are found, in the usual way, by evaluating the downwash at each point, equating it to the value given by the boundary condition and solving the resultant system of linear equations. The wing aerodynamic coefficients are then found from the results, to a linear approximation.

A first approximation to the rolling up is now found by taking the actual spanwise distribution of the circulation (which will generally not be elliptic) and performing a two-dimensional calculation of rolling up, as was done in Section (a) for the elliptic case. This is now taken as the first approximation to the rolled up wake shape starting from the trailing edge. As in Section (b), the first approximation to the wake shape is improved iteratively using the calculated vortex strengths. During these iterations the vortex strength on the wing are kept constant. Using the new wake shape

The system of bound-vortex elements is set up exactly as in the previous section and the vortex strengths and wing properties are found by a similar process. The rolling up of the vortex wake can be obtained by calculating the induced motion of the individual vortex lines due to mutual interactions. The final shape is determined by an iteration procedure described in Fig. 3, involving the recalculation of the vortices strengths and the corresponding wake shape. This calculation cannot be done using the ideal line vortices because of the difficulties of intersection and "escape" of the vortices discussed previously. Therefore, this model is used only to determine the nonlinear aerodynamic coefficients with straight trailing vortices. The use of finite core vortices may eliminate some of the problems and enable the calculation of this rolled up vortex wake. This calculation is now underway. However a simpler calculation is possible when it is assumed that vortices are shed only fron the planform edges. In this case the new vortex wake shape can be recalculated by the iterative procedure indicated in Fig. 4 step 1.

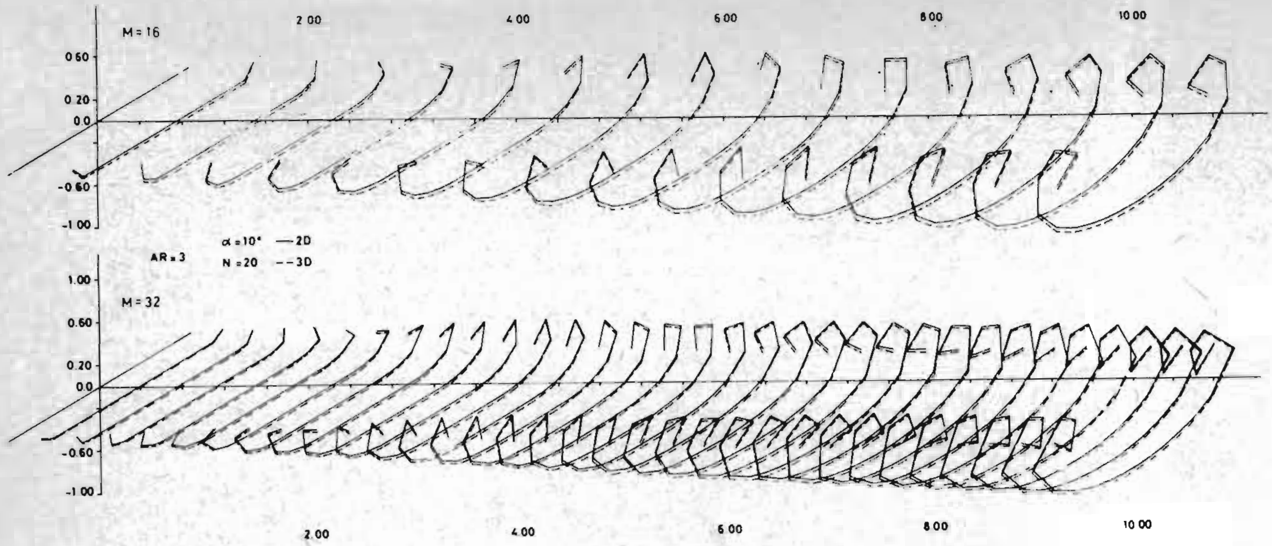

Figure 9. The Rolled-up Wake Shapes Calculated by the VLM Procedure with Step Sizes M = 16 and M = 32.

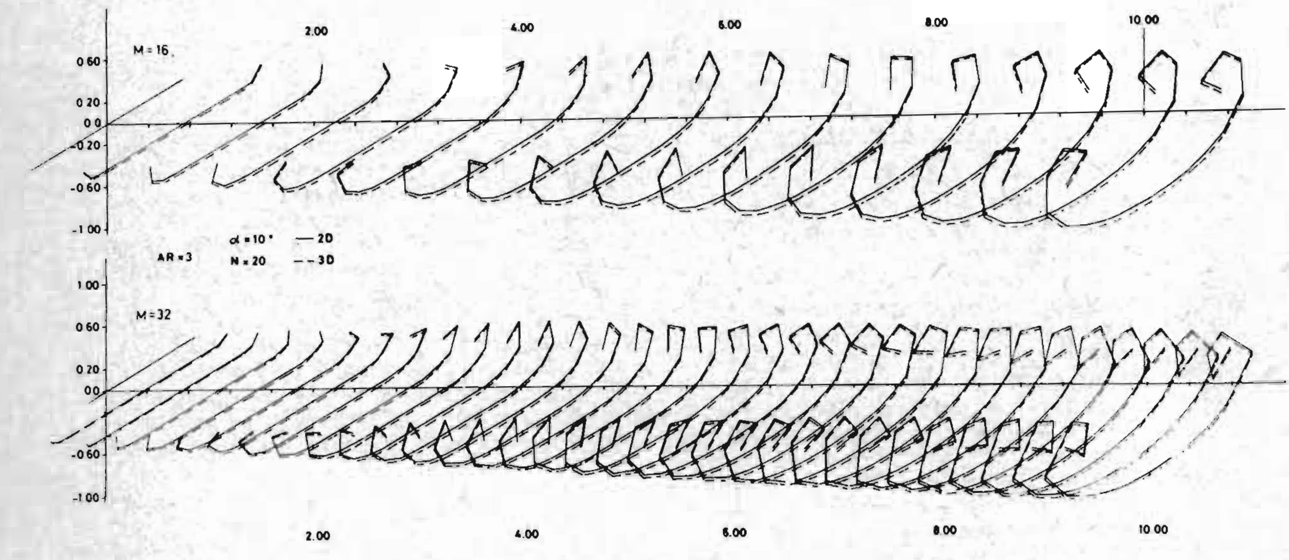

Figure 10. The Rolled-up Wake Shapes Calculated by the MVLM Procedure with Step Size M = 16 and M = 32.

IV. The Finite Core Vortices

The numerical problems associated with the use of ideal line vortices lead us to look for a solution by the use of finite core vortices. Since the actual wake has thickness and is not an infinitesimally thin sheet, a better approximation to the real wake is therefore an array of vortices with cores of solid body rotation. We assume a core diameter which is determined by the condition of constant vorticity per unit cross-sectional area of the core. The density of the vorticity in the core can be calculated, for instance, by using the core size calculated by Spreiter and Sacks[19]. We impose the condition that as soon as two cores touch the two vortices are replaced by a single one at the center of gravity of the pair. The new core diameter is determined by the condition of conservation of vorticity at the same vortex density. This procedure overcomes the difficulties caused by the approach of ideal vortices, i.e. "intersection" and "escape". This procedure limits the number of vortices automatically, thus helping to eliminate also other irregularities. Furthermore, this method ensures that, when all the vortices are amalgamated far downstream, we end up with a pair of vortices with the correct diameter and strength matching the expected vortex pair in the far field.

Under these assumptions the core radius is determined by the relation,

$$R_i^2 = \frac{K_i}{\sum\limits_{j=1}^{n} K_j} \cdot R_o^2 \qquad (13)$$

R_o being the final core diameter.

The vortex core diameter is evaluated at present by using the method presented by Spreiter and Sacks[19]. Accordingly the core diameter is

$$\frac{R_o}{b'} = 2\{1+\exp[\frac{4\pi \cdot AR. C_{D_i}}{C_L^2}(\frac{b'}{b})^2 - \frac{1}{4}]\}^{-1} \qquad (14)$$

when the wing loading is elliptical, $C_{D_i} = C_L^2/\pi AR$ and $b'/b = \pi/4$. In this case the core radius of the vortex in the far field is $R_o = 0.197b' = 0.155b$.

The vortex wake shape obtained when originally 41 finite core vortices are equally spaced on the trailing edge is shown in Fig. 11. It should be pointed out that during the wake development calculations a number of vortices combine and we find in the wake less vortices than the initial number. In addition the vortex sheet shows the typical "kink" found by Westwater.

The procedure for the calculations of the wake characteristics associated with the finite core vortices are outlined in Fig. 4. The detailed procedure for the calculations of the vortex core diameter is outlined in Fig. 4a.

V. Results of the Calculations of Aerodynamic Characteristics.

The various method presented in this report can be used to calculate the aerodynamic characteristics of various wings. Using models of increasing complexity it is possible to proceed from the classical VLM calculation of the linear aerodynamic coefficients to calculations resulting in non-linear aerodynamic coefficients.

Firstly, we present results based on VLM calculations for delta wings. The lift coefficient for delta wings at $\alpha = 10^o$ as a function of the Aspect Ratio is shown in Fig. 12. The next step was to calculate the lift coefficient using the Gersten model, i.e. free straight vortices shed at an angle $\alpha/2$ from each cell. The appropriate results are also indicated in Fig. 12. The computer programs written for all of these calculations are presented in Ref. (33). The non-linear lift variation for a delta wing of AR = 0.5 is presented in Fig. 13, and for a swept back wing of AR = 1 in Fig. 14. In both cases it is found that the calculation based even on this simplified Gersten model for the vortex wake result in agreement with the experimental results.

Similar calculations are performed for rectangular wings. The calculated lift coefficients at $\alpha = 10^o$ for the rectangular wings of varying aspect ratio is shown in Fig. 15. The non-linear program is used to evaluate the lift coefficient of a narrow rectangular wing of AR = 1/30, which was first investigated by Bollay[3]. The results are shown in Fig. 16, (included are experimental results up to $\alpha = 35^o$). It can be seen that the lift variation predicted by the non-linear program based on the simplified Gersten model does follow the experimental data over the complete range (in the case of $N_c = 2$, $N_s = 30$).

The success of this simplified non-linear model to predict the lift coefficient variation with angle of attack obtained in these calculations must be further examined. In order to do so, the aerodynamic characteristics of a rectangular wing of AR = 1 are calculated based on the non-linear vortex lattice procedures increasing in complexity, as described in this report. The aerodynamic characteristics of this wing are presented in Table I. This Table presents the results of the calculations of C_L, C_M, $x_{c.p.}$, C_{D_i} and C_{D_i}/C_L^2 obtained by the following methods: (1) the non-linear VLM using Gersten's model, (2) straight line vortices trailing off the side and trailing edges, (3) vortices trailing off the side and trailing edge including effects of the rolling up. The variation of C_L, C_M and C_{D_i} as a function α obtained in these

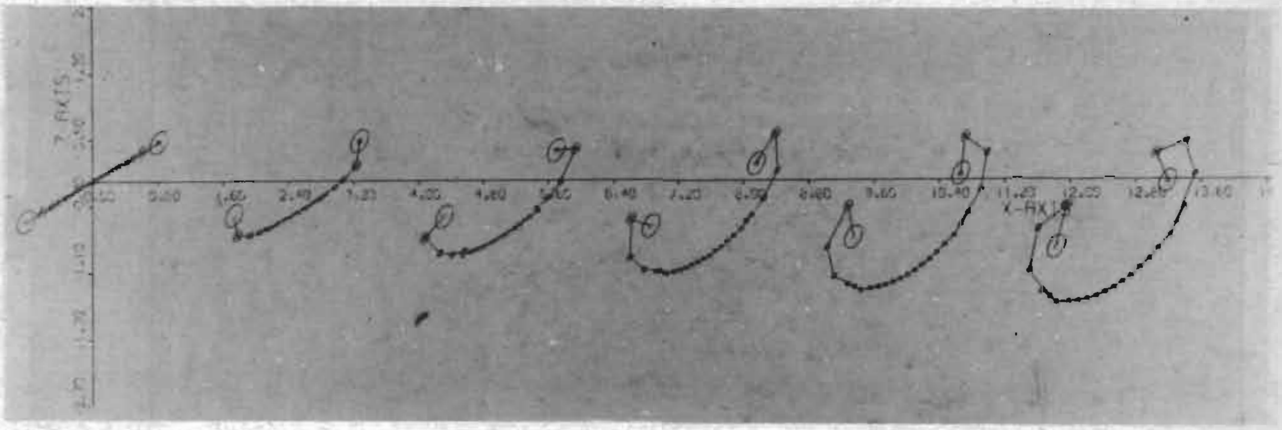

Figure 11. The Rolled up Wake Shape Calculated by the VLM Procedure with Finite Core Vortices. Rectangular Wing of AR = 1 at $\alpha = 10^o$ and N = 41, M = 32.

calculations are presented in Figs. 17, 18 and 19 respectively. Included are some experimental results. It can be seen that the values of C_L and C_M determined by the various methods used in these calculations are rather close and are all reasonably close to the experimental data. A larger variance between the results of the various calculations is found in the value of C_{D_i}. Furthermore these calculated values of C_{D_i} are all somewhat lower than the measured values.

The calculated pattern of the vortices over the wing and in the near wake is shown in Fig. 20. The rolling up of the vortex sheet over the wing due to the distinct side vortices is clearly seen.

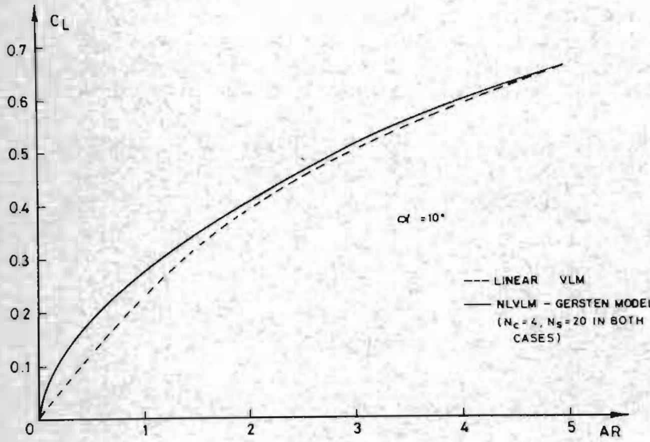

Figure 12. Comparison of the Linear Lift Co-Efficient Calculated by the VLM Program and the Non-Linear Lift Coefficient by the NLVLM Program for delta wings at α = 10°.

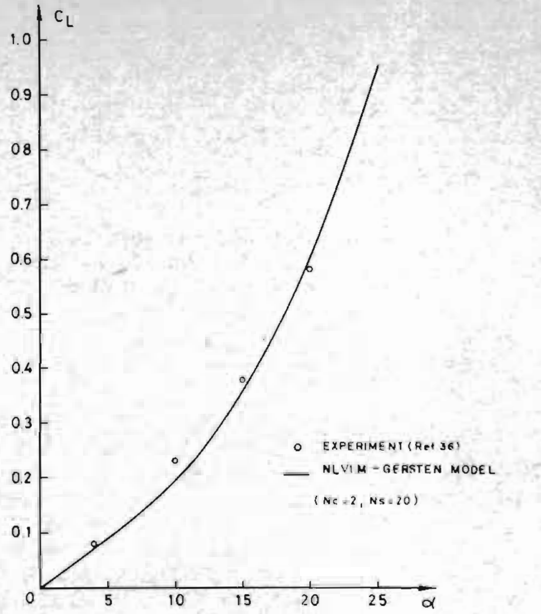

Figure 13. The Lift Coefficient of a Delta Wing of AR = 0.5.

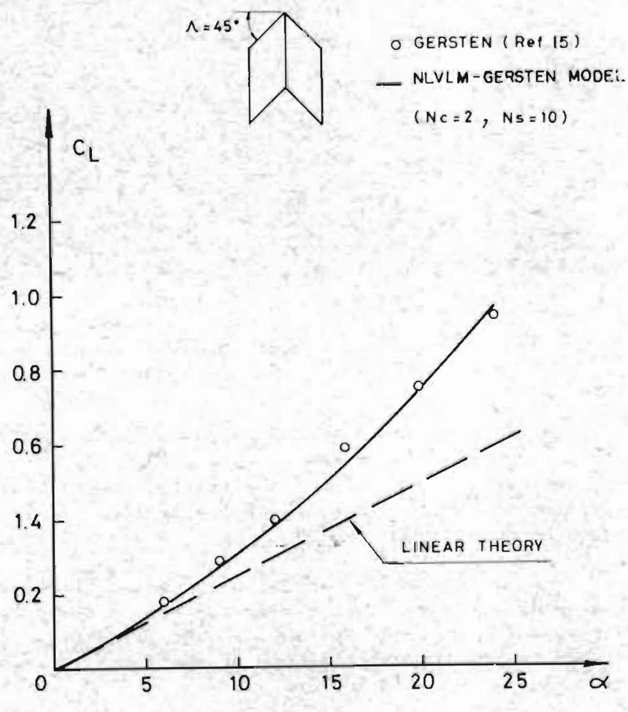

Figure 14. The Lift Coefficient of a Swept Back Rectangular Wing of AR = 1.

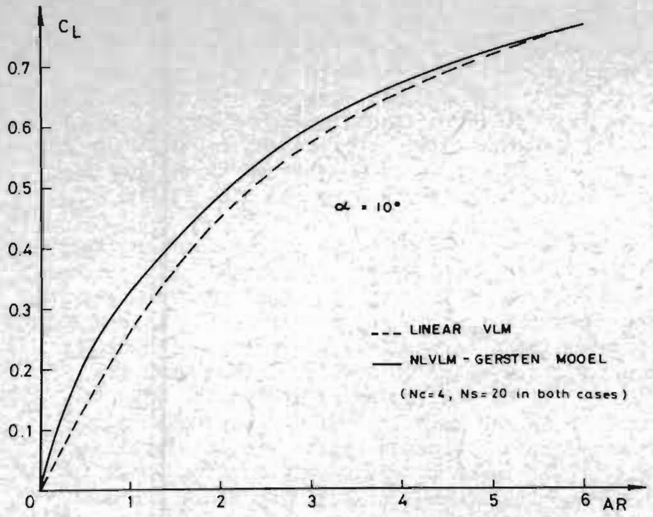

Figure 15. Comparison of the Linear Lift Coefficient Calculated by the VLM Program and the Non-Linear Lift Coefficient Calculated by the NLVLM Program for Rectangular Wings at $\alpha = 10^{o}$.

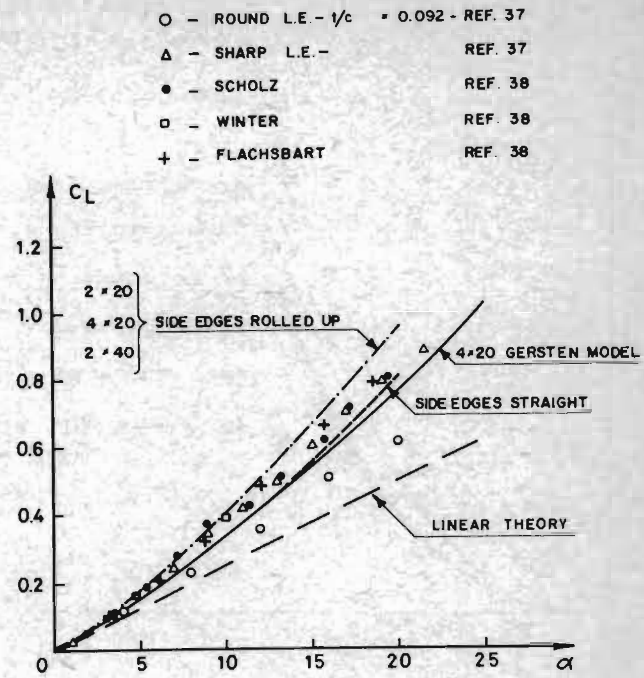

Figure 17. The Lift Coefficient of a Rectangular Wing of AR = 1.

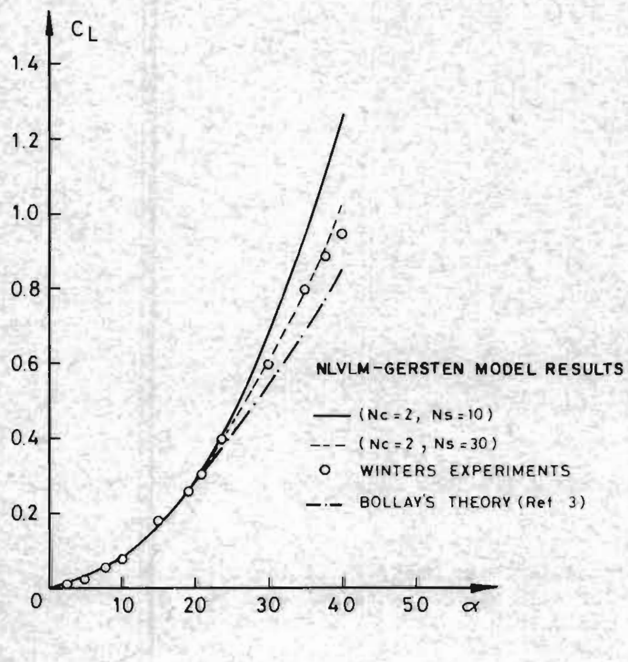

Figure 16. The Lift Coefficient Calculated by the NLVLM Program compared with the Results Presented in Ref. 13, for Rectangular Wing of AR = 1/30.

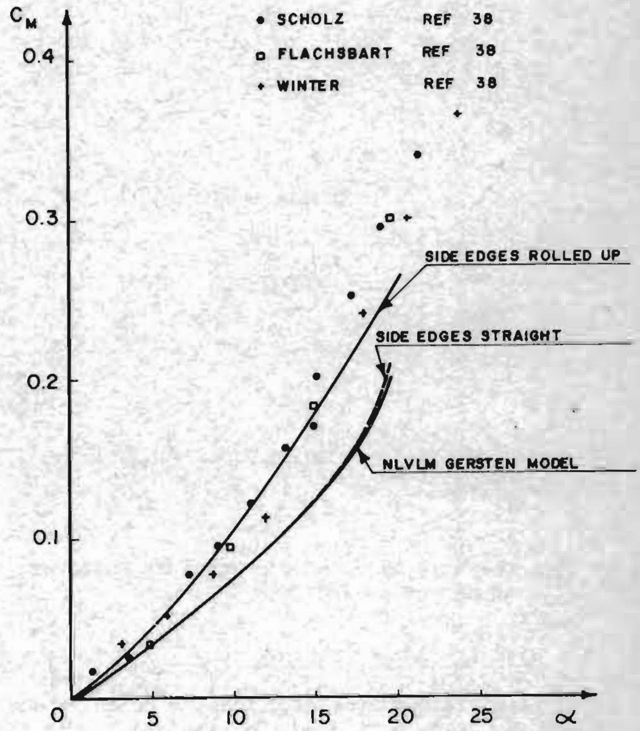

Figure 18. The Pitching Moment Coefficient of a Rectangular Wing of AR = 1.

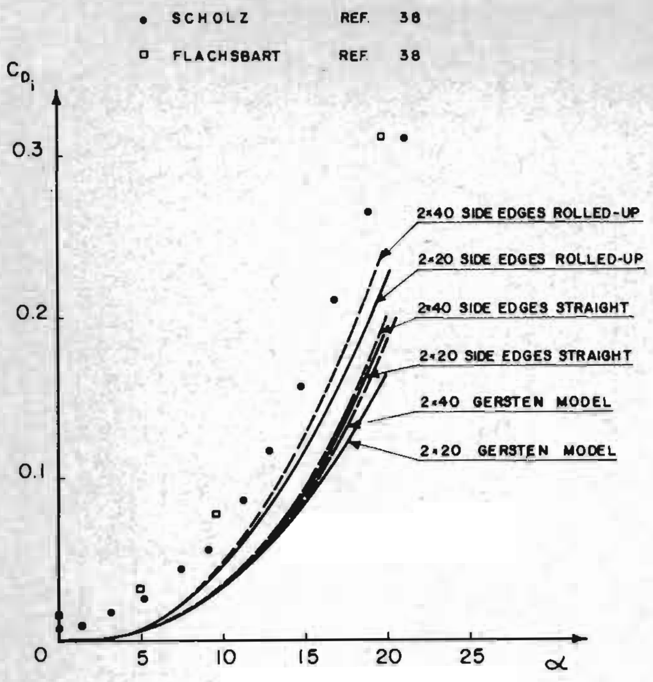

Figure 19. The Induced Drag Coefficient of a Rectangular wing of AR = 1.

the loading of each cell is represented by a bound vortex and a pair of trailing vortices which can be either constrained to stay in the wing planform until the trailing edge (VLM procedure) or be allowed to leave the wing locally either at a fixed angle (as in Gersten's model) or these free vortices can be allowed to follow unconstrained trajectories which are then determined by the flow field.

It is expected that the calculations using the Vortex Lattice Method, with the trailing vortices of every cell remaining in the wings' planform and shed away from the trailing edge only result in evaluating the linear aerodynamic characteristics of the wing. It was found in Ref. (33) that when the effects of the rolling up of the vortex wake are included in the calculations the evaluated aerodynamic characteristics remain essentially linear. This result is obtained in calculations which follow the computing scheme indicated in Figs. 1 and 2 for wings of small and large aspect ratios.

It is found that non-linear aerodynamic characteristics are obtained only when free vortices are placed above the wings' planform. This can be obtained when the trailing vortices from each cell are allowed to leave the wings' planform at either a fixed angle (Gersten's model) or can be allowed to follow unconstrained trajectories over the wing to be determined by the iterative calculations. In the case of free vortices at a fixed angle the value suggested by

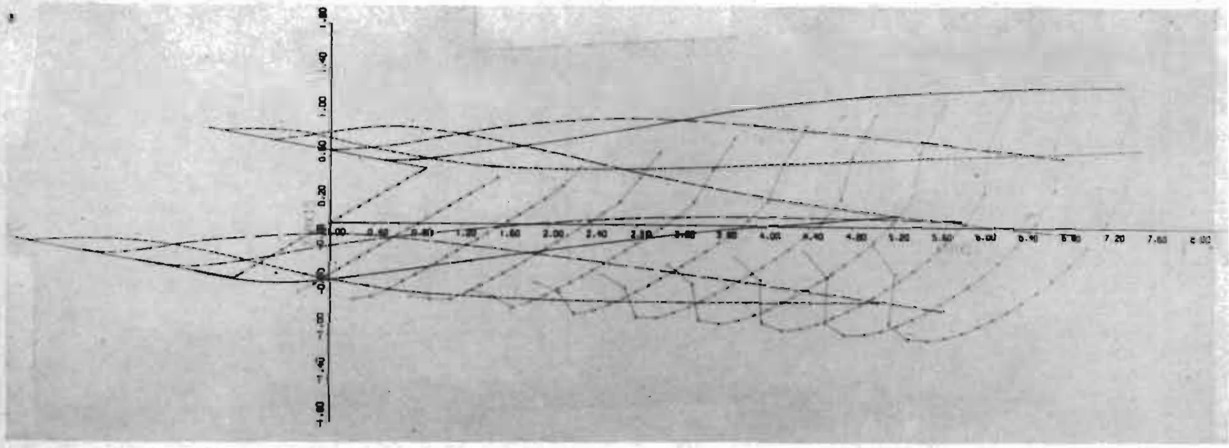

Figure 20. The Rolling Up of the Vortices Shed from the Side and Trailing Edges over a Rectangular Wing of AR = 1 at $\alpha = 10^{o}$.

VI. Discussion and Conclusions

The various procedures used for the calculations of the loading distribution on the wing and the trailing vortices, are based on the approximation of the distributed vorticity by discrete vortices. These discrete vortices are distributed over the wing planform and are allowed to leave the planform into the wake under various constraints. The present investigation presents an attempt at a systematic study of the possible vortex systems that can be used in the calculation schemes.

The first step in all of these approximations is the subdivision of the wing planform in the chordwise and spanwise directions into many cells. Now Gersten ($\alpha/2$) is used. It is found that the non-linear lift coefficient evaluated by this method is reasonably close to the experimental results as seen in Figs. 13,14,16 and 17. However the pitching moment and particularly the induced drag evaluated by this method do not agree as well with the experimentally determined values (Figs.18 and 19).

The problems associated with using discrete ideal vortices make it practically impossible to calculate the trajectories of the interacting free vortices shed from each cell. Therefore, a model of vortex shedding was proposed where free trailing vortices are allowed to leave the wings' planform only at the side edges and the trailing edges.

Therefore the trailing vortices from each cell are constrained to stay in the wings' planform except those vortices of the cells along the side edges. In this calculation the free vortices from the side edges are first assumed to leave at the fixed angle of $\alpha/2$ and remain straight. The results of the calculation using this model are compared in Table I with the previous calculations based on the non-linear Gersten model. It is seen that the differences between these results are rather small. This comparison justifies the assumption that the main non-linear effects can be attributed to the contributions of the vortices shed from the side edges.

A more realistic model can be obtained by allowing the vortices from the side edges to interact and roll over the wing and its rolled up wake. In this case, as shown in Fig. 4, the trajectories of the vortices leaving from the side edges can be calculated and the corresponding aerodynamic characteristics be determined. An example of the trajectories of these vortices is shown in Fig. 20. The corresponding aerodynamic characteristics for the rectangular wing of AR = 1 are listed in Table I and indicated in Figs. 17, 18 and 19. The results obtained by this calculation show that the lift coefficient is increased by the effect of the rolling up of the side vortices on the wing. The newly calculated lift coefficients are in good agreement with the experimental results as seen in Fig. 17. Even more significant improvements are obtained in the calculation of the pitching moment, Fig. 18. The evaluation of the induced drag is also improved, Fig. 19, although the calculated induced drag is somewhat lower than the experimental values. It is seen that the effects of rolling up of the side edges vortices is to increase the induced drag. The effects of chordwise subdivisions are small. The effects of spanwise subdivisions are also quite small as seen in these Figures and in Table I. Some of the additional drag may be attributed to the fact that the experimental results present the total drag while the calculated value only the induced drag. However, the remaining discrepancy may be due to the nature of the approximation of the vortex model used.

We expect to be able to obtain more realistic models by replacing the discrete ideal vortices by finite core vortices. Physically we argue that the finite core vortices are a better representation of the real vortex wake of a wing (which is always of a finite thickness due to the effects of viscosity).

Replacing the ideal vortices by the finite core one does eliminate the problems associated with the numerical problems associated with the interactions of vortices. Some preliminary calculations do indicate that such calculation scheme is feasible. The result presented in Fig.11 is certainly encouraging. This finite core vortex model can be now applied to the more complex cases where the vortices are allowed to leave the wings' planform either at the side and trailing edges or even for the case of free vortices from each cell. Such calculations are now in progress.

Acknowledgement

The authors wish to acknowledge the suggestions and the fruitful discussions with Professor H. Portnoy during the course of this study.

Table I The Calculated Aerodynamic Coefficients for a Rectangular Wing AR = 1.

Method	Subdivisions $N_c \times N_s$	α	C_L	C_M	$x_{c.p.}$	C_{D_i}	C_{D_i}/C_L^2
Non-Linear VLM-Gersten Model	2 x 20	5	0.148	0.0326	0.22	0.0064	0.295
		10	0.329	0.0776	0.236	0.0306	0.284
		15	0.54	0.1330	0.246	0.0805	0.276
		20	0.779	0.198	0.254	0.164	0.270
	2 x 40	5	0.148	0.0327	0.221	0.0066	0.301
		10	0.332	0.0780	0.235	0.0329	0.299
		15	0.547	0.1350	0.246	0.0899	0.301
		20	0.792	0.201	0.254	0.188	0.299
Vortices Shed from Side and Trailing Edges-Straight Lines	2 x 20	5	0.144	0.0308	0.214	0.00619	0.298
		10	0.321	0.0738	0.230	0.0300	0.292
		15	0.54	0.131	0.242	0.0839	0.287
		20	0.809	0.204	0.252	0.182	0.278
	2 x 40	5	0.143	0.0306	0.214	0.00631	0.307
		10	0.325	0.0749	0.23	0.0329	0.311
		15	0.551	0.134	0.243	0.0922	0.304
		20	0.825	0.209	0.253	0.195	0.287
	4 x 20	5	0.146	0.0294	0.202	0.0063	0.296
		10	0.323	0.0721	0.223	0.0296	0.283
		15	0.536	0.128	0.239	0.0773	0.269
		20	0.791	0.199	0.252	0.160	0.255
Vortices Shed from Side and Trailing Edges-Rolled Up	2 x 20	5	0.169	0.0412	0.244	0.00834	0.293
		10	0.391	0.101	0.259	0.0438	0.286
		15	0.676	0.178	0.264	0.122	0.267
		20	0.935	0.256	0.274	0.221	0.253
	2 x 40	5	0.172	0.0414	0.241	0.00941	0.320
		10	0.398	0.103	0.259	0.0478	0.301
		15	0.665	0.176	0.264	0.124	0.279
		20	0.969	0.259	0.267	0.240	0.255
	4 x 20	5	0.177	0.0429	0.242	0.0086	0.275
		10	0.395	0.0980	0.248	0.0397	0.255
		15	0.665	0.174	0.262	0.104	0.235
		20	0.951	0.252	0.265	0.198	0.219

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DISCUSSION

D. Hummel (Technical University, Braunschweig, Germany): I would like to comment on your conclusions concerning the aerodynamic coefficients of wings, the free vortices of which originate from the trailing-edge only. For this case you have calculated the shape of the wake down-stream of the wing. It turns out that the wake position relative to the wing is a function of the angle of incidence. In this case you must have a nonlinear dependence of the aerodynamic coefficients on the angle of incidence. The nonlinear parts of the aerodynamic coefficients may be very small but they do exist.

J. Rom, C. Zorea and R. Gordon: Professor Hummel's remark is basically correct. However, we find that the effect of the vortices downstream of the trailing edge of the wing on the circulation distribution on the wings' surface is very small. This is the reason for the very good prediction of the linear aerodynamic coefficients using any of the known methods based on the evaluation of the lift integral or on the vortex lattice methods which neglect the rolling of the trailing vortices. We find that "noticeable" nonlinear effects are produced only when free vortices are shed above the wings' surface, that is on the wings' edges or surface ahead of the trailing edge.

A. Das (DFVLR-Braunschweig, Germany): Your paper contains a very detailed analysis of the non-linear aerodynamic characteristics of wings based on various models for the vortex wakes. It comes out that the vortex model after Gersten and the one with plane vortex sheet containing only straight inclined edge vortices produce the same lift forces on the wing. However, it is to be expected that the spanwise and chordwise distribution of circulation or induced velocities would be quite different in the two cases. A plot of lift or circulation over the span length and of pressure distributions along the chords would give more insight into this problem. This will also reveal how much these vortex models fail to reproduce the flow condition on the whole wing surface as compared to the real case with rolled up vortex sheet.

J. Rom, C. Zorea and R. Gordon: The use of straight line vortices in the Gersten model and in the model assuming vortex shedding only from the edges results in essentially the same lift because the vortices inside the planform are rather weak. There is some difference in prediction of the pitching moment and the induced drag showing that indeed the load distribution is different for these models.

It is possible to obtain the load distribution for each case using our present computer programs and we plan to compare the load distributions on the wings as suggested by Dr. Das.

B.G. Newman (McGill University, Montreal, Canada): I should like to congratulate the authors on their prediction of, in particular, pitching moment. I have two questions :

1. Since it is strictly incorrect to apply the Biot-Savart law to regions containing vorticity, how do you calculate the movement of the finite cores?
2. Does the size of the core change downstream?

J. Rom, C. Zorea and R. Gordon:

1. The finite core model assumes that the vortices have a core of constant angular momentum density and outside of this finite size core the flow field corresponds to that of the ideal inviscid vortex. Therefore the Biot-Savart law can be applied over the complete flow field outside the cores.
2. In the present model the core size is determined by the assumption of constant angular momentum density. Therefore the core size can change only by amalgamation of vortices when their cores intersect. That is the core size remains constant as long as there is no intersection with another vortex core.